Are pairings really dead? ☠

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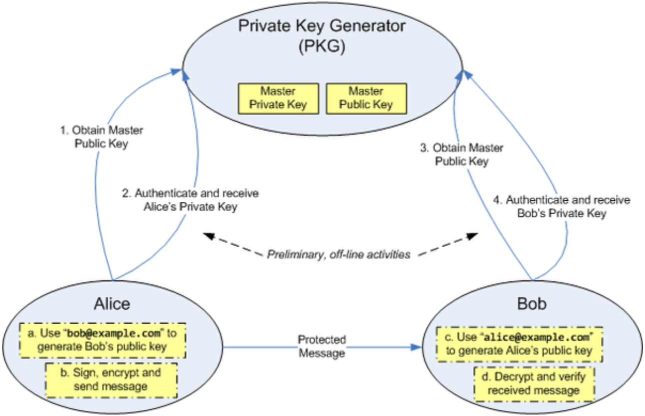

Image: Identity-based encryption; stolen shamelessly from Wikipedia

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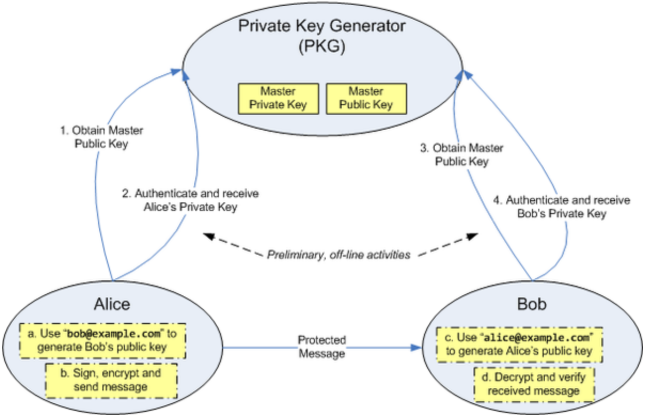
- Allows for anonymous authentication. How?



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  - eg. $(3 \pmod 5)^2 = 3 \cdot 3 \pmod 5$.

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Why is this useful?

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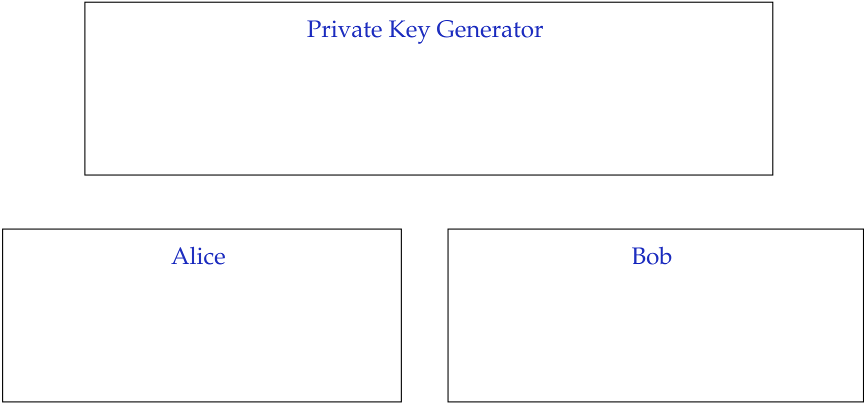
Scenario: Bob authenticates an anonymous Alice.



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  - Instances of the Weil pairing can be efficiently computed with Miller's algorithm.

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for most pairing instances, p is special, giving $L_{p^k}(1/3, c)$

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$O(\sqrt{r})$, where r is the largest prime dividing $\#E(\mathbb{F}_{p^k})$

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- With new understanding, different parameters will rule them all.

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3 concrete approaches so far:

- Just increase the parameters for BN and BLS until they are secure [BD16].

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Why is this not a 'concrete approach'?

- Concrete security level not yet calculated.
- Concrete timings not yet integrated.
- May be a faster candidate, but currently unknown!

So where are we at?

The computation of a pairing like those above can be boiled down to multiplications in $\mathbb{F}_p$, where $\mathbb{G}_3 = \mathbb{F}_{p^k}^*$.

$\mathbf{m}$ = one $\mathbb{F}_p$-multiplication.

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Pairing choice	$\log(p)$	Pairing cost	Clock cycles
BN	462	17871$\mathbf{m}$	2966586
$k = 6$ [GMT]	672	8472$\mathbf{m}$	2660208
KSS	339	25926$\mathbf{m}$	2566674
$k = 8$ [GMT]	544	11636$\mathbf{m}$	2443560
BLS	461	13878$\mathbf{m}$	2303748
Family 17a [FM]	398	16189$\mathbf{m}$	2088381
Family 17b [FM]	407	16172$\mathbf{m}$	2086188

Table: Choices for 128-bit security

The number of clock cycles is based on a generic Montgomery-schoolbook algorithm for multiplication mod p on a 64-bit processor.

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Thank you!

References

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