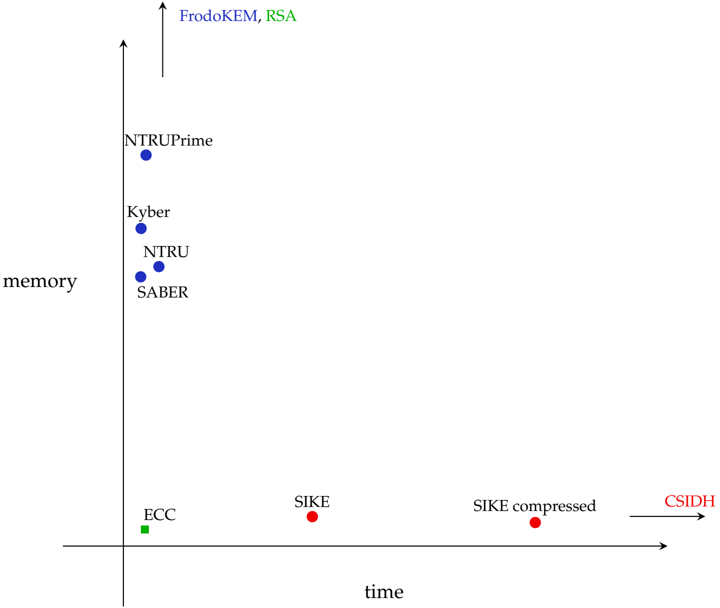
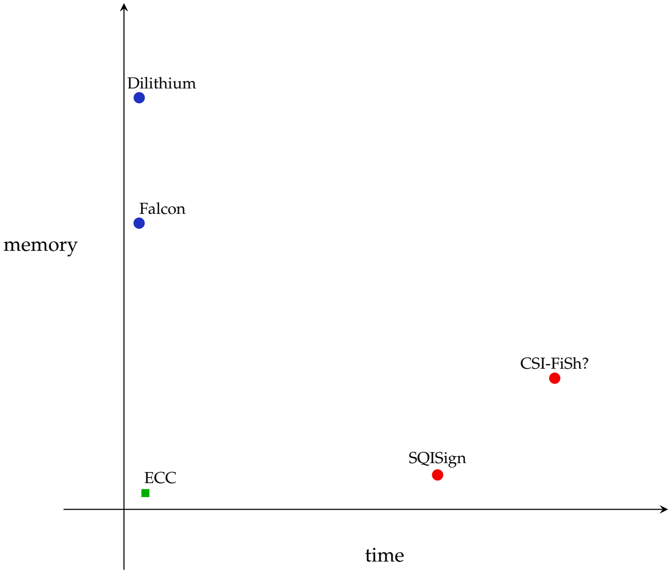
Isogeny-based cryptography: why, how, and what next?

29th July 2022

Zoo of lattice- and isogeny-based KEMs



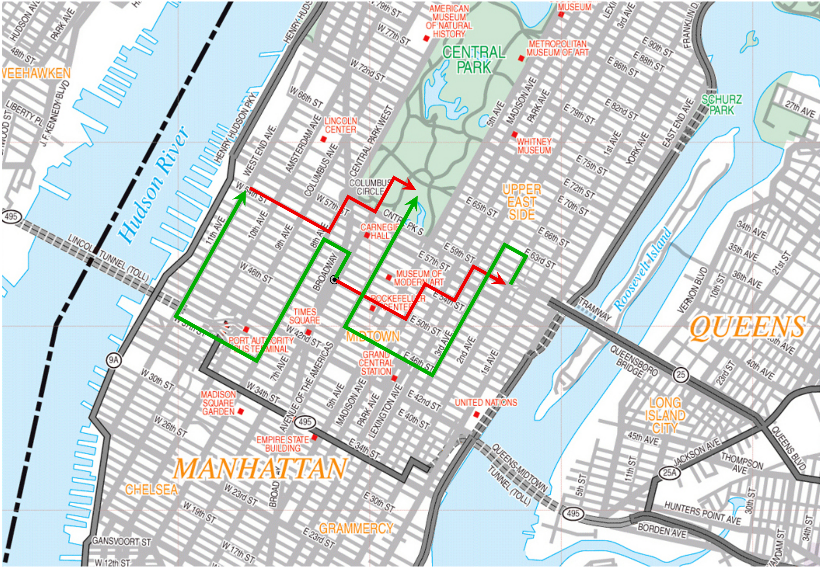
Zoo of lattice- and isogeny-based signatures



Applications (non-exhaustive list)

	Lattices	Isogenies
KEM	✓	✓
Signatures	✓	✓
NIKE	(×)	✓
FHE	✓	×
IBE	✓	×
Threshold	✓	✓
OPRF	✓	✓
VDF	(×)	(✓)
VRF	(✓)	(✓)

Graph walking Diffie–Hellman?



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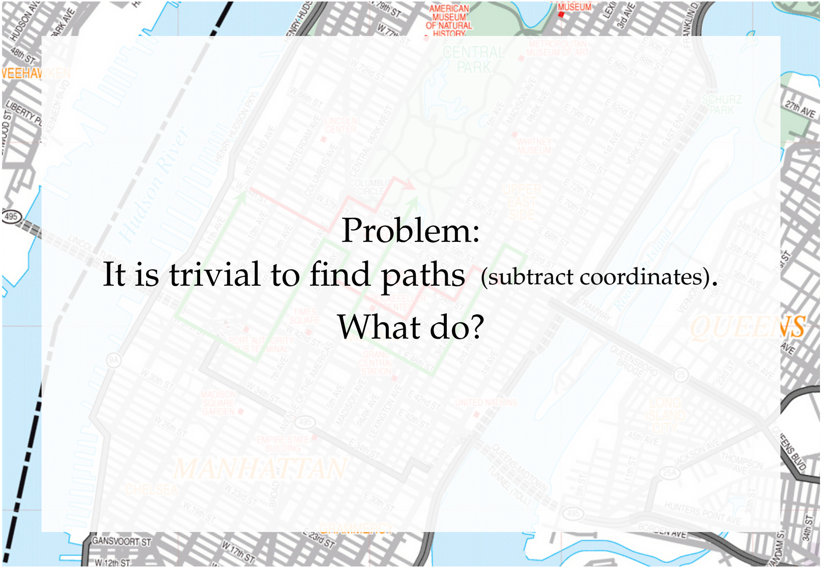

Problem:

It is trivial to find paths (subtract coordinates).

What do?

Big picture

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It is easy to construct graphs that satisfy *almost* all of these — not enough for crypto!

Stand back!

We're going to do maths.

The beauty and the beast

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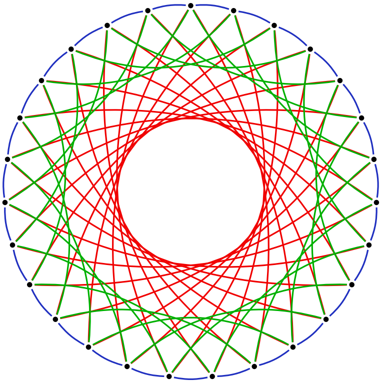

$S = \{3, 5, 7\}$, $q = 419$

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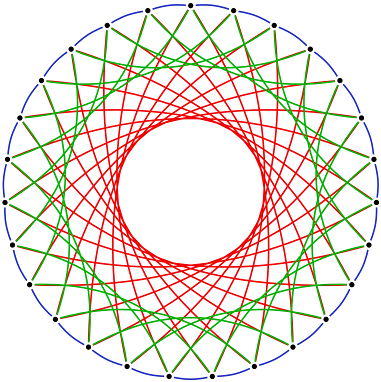

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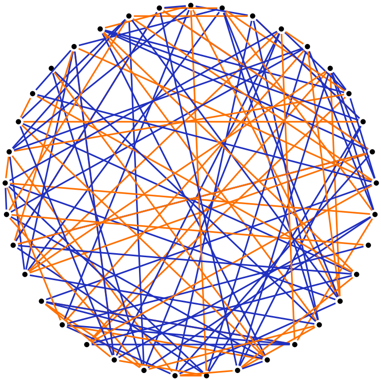
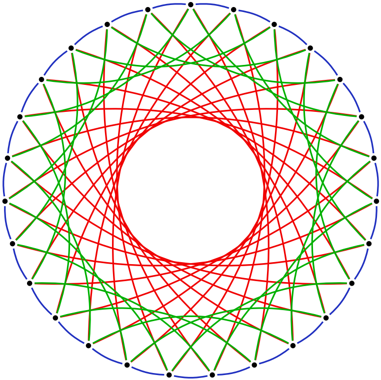


$S = \{2, 3\}$, $q = 431^2$

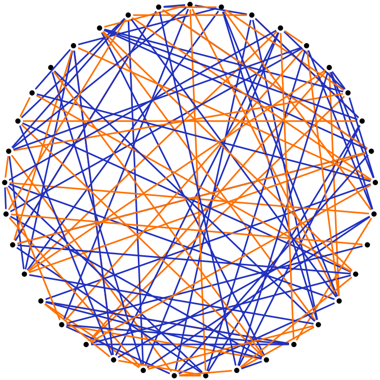
The beauty and the beast

For key exchange/KEM, there are <u>two families</u> of systems:



$q = p$

CSIDH [ˈsiːˌsaɪd]
https://csidh.isogeny.org



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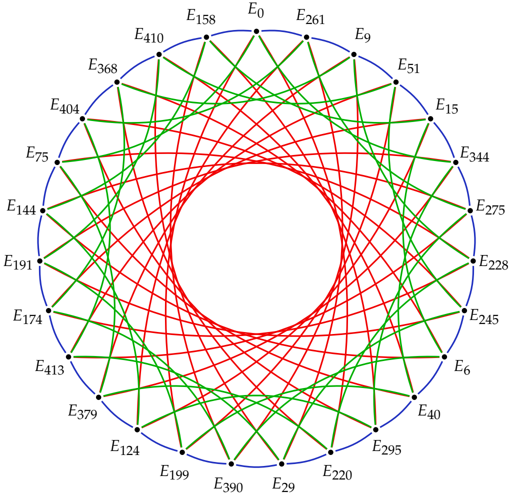
SIDH
https://sike.org


[ˈsiːˌsaɪd]

Isogeny graphs at the CSIDH



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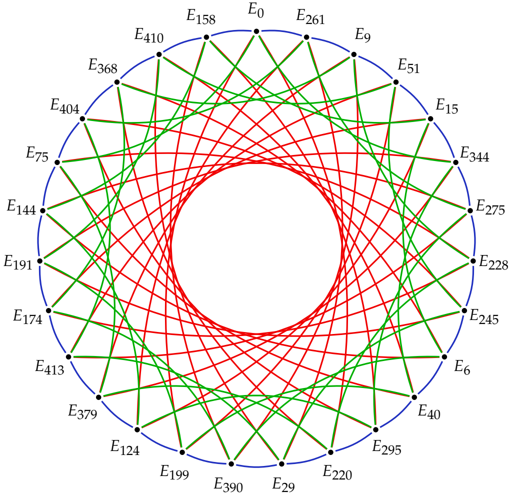

Nodes: Supersingular curves $E_A : y^2 = x^3 + Ax^2 + x$ over $\mathbb{F}_{419}$.

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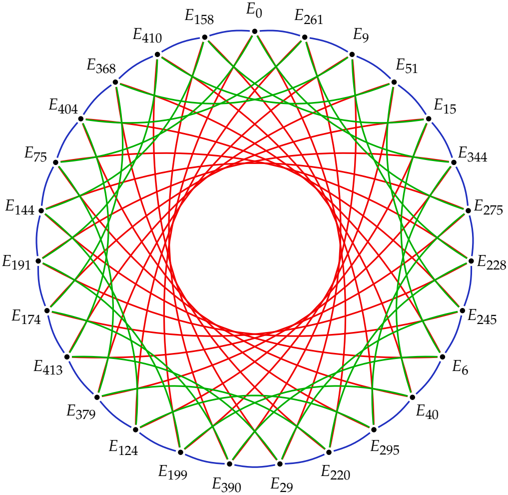


Nodes: Supersingular curves $E_A : y^2 = x^3 + Ax^2 + x$ over $\mathbb{F}_{419}$.
Edges: 3-, 5-, and 7-isogenies.

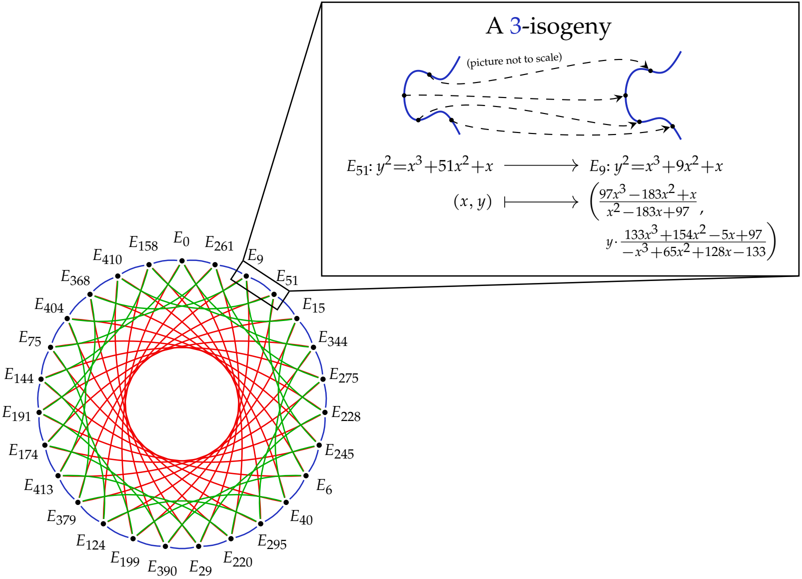
Quantumifying Exponentiation

- Idea to replace DLP: replace exponentiation

$$\begin{array}{rcl} \mathbb{Z} \times G & \to & G \\ (x, g) & \mapsto & g^x \end{array}$$

  by a group action on a set.
- Replace G by the set S of supersingular elliptic curves $E_A : y^2 = x^3 + Ax^2 + x$ over $\mathbb{F}_{419}$.
- Replace $\mathbb{Z}$ by a commutative group H that acts via isogenies.
- The action of $h \in H$ on S moves the elliptic curves one step around one of the cycles.

Graphs of elliptic curves



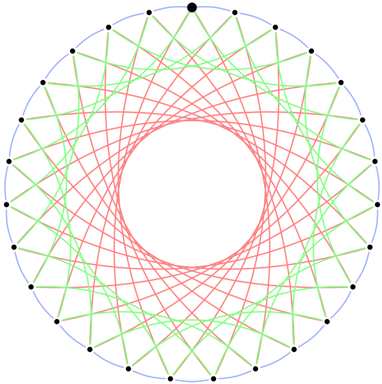
$$E_{51}\colon y^2 = x^3 + 51x^2 + x \longrightarrow E_9\colon y^2 = x^3 + 9x^2 + x$$

$$(x, y) \longmapsto \left(\frac{97x^3 - 183x^2 + x}{x^2 - 183x + 97},\ y \cdot \frac{133x^3 + 154x^2 - 5x + 97}{-x^3 + 65x^2 + 128x - 133} \right)$$

Diffie and Hellman go to the CSIDH

Alice

$[+, -, +, -]$



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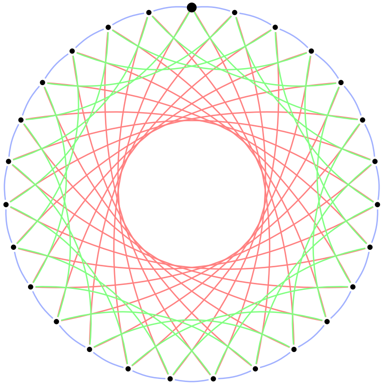

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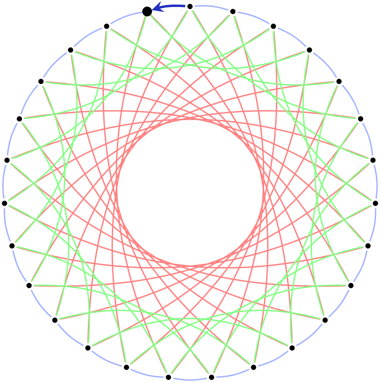

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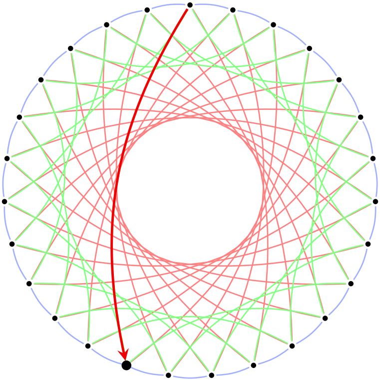

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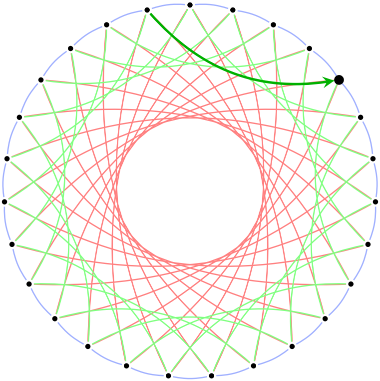

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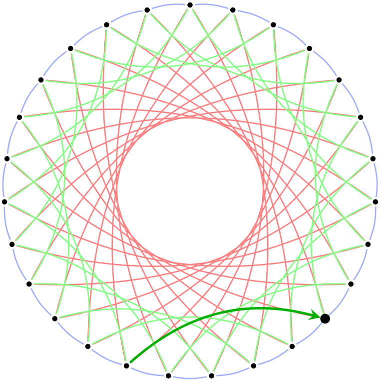

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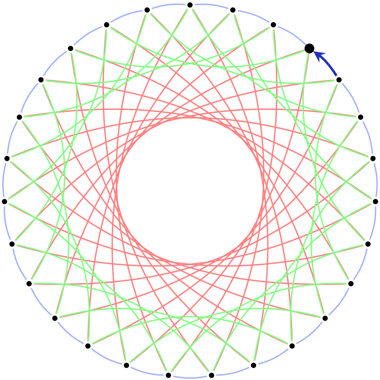

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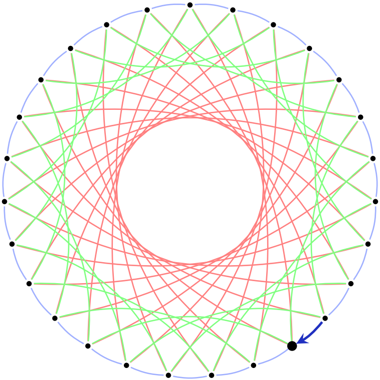

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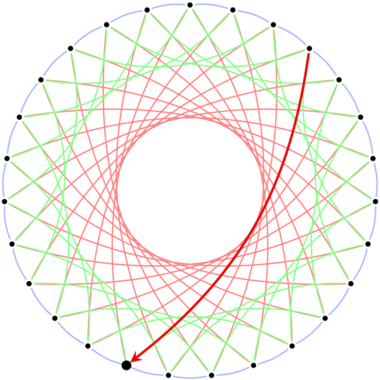

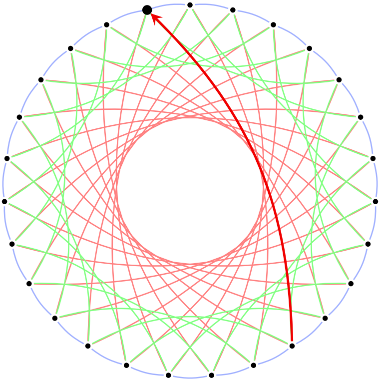

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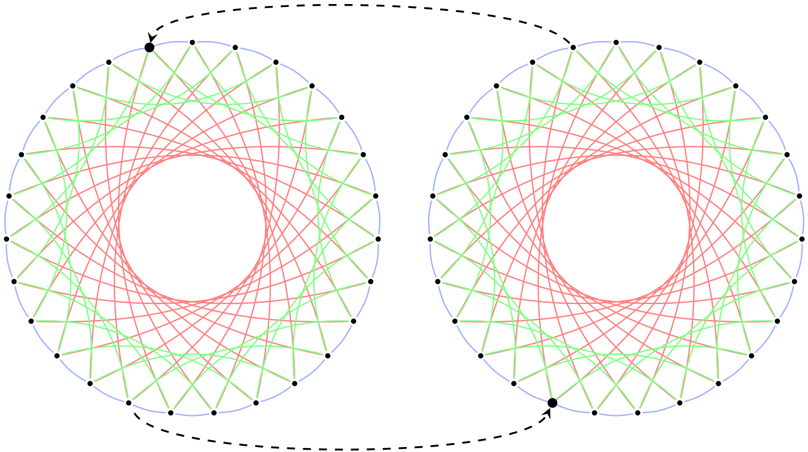

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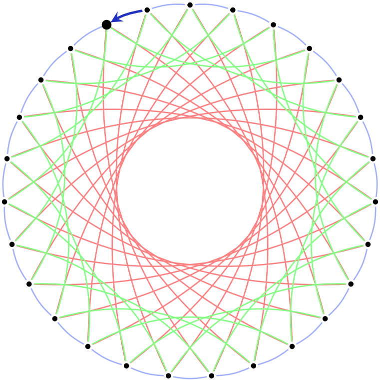

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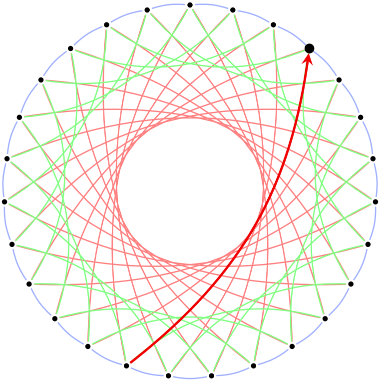

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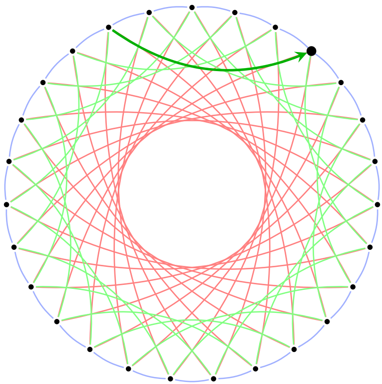

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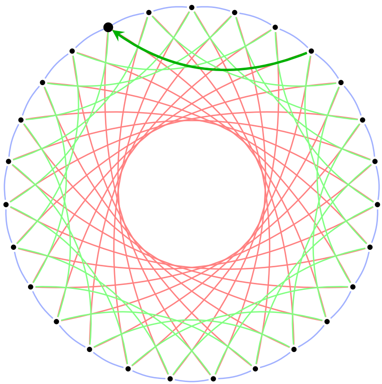

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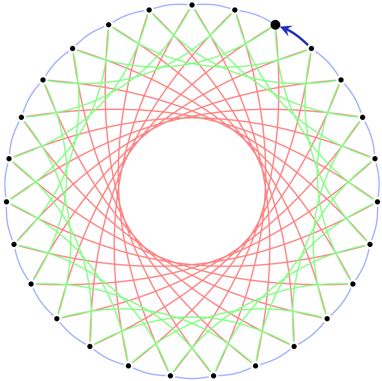

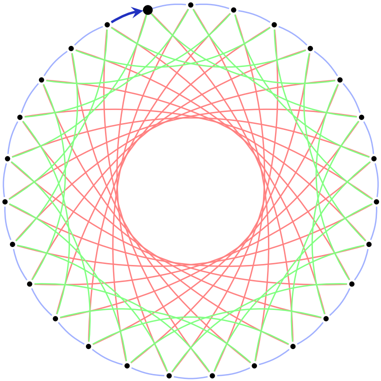

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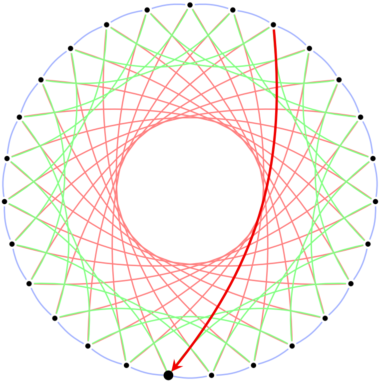

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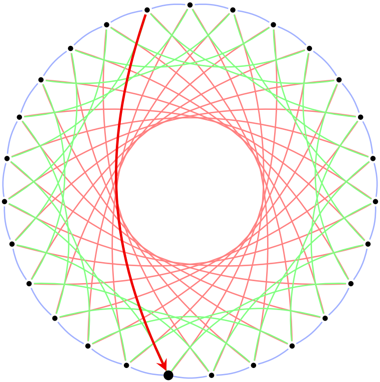

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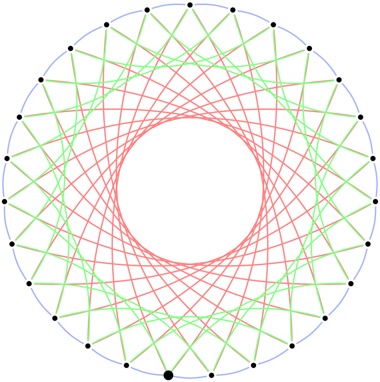

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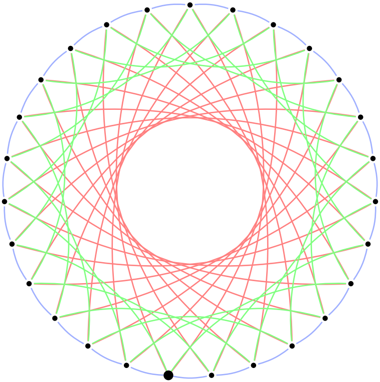

Compute neighbours in the graph

To compute a neighbour of E, we have to compute an ℓ-isogeny from E. To do this:

- Find a point P of order ℓ on E.

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  - Given a $\mathbb{F}_p$-rational point of order ℓ, the isogeny computations can be done over $\mathbb{F}_p$.

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$\Rightarrow$ Tiny keys!

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No.

- About $\sqrt{p}$ of all $A \in \mathbb{F}_p$ are valid keys.
- Public-key validation: Check that E_A has $p + 1$ points.

  Easy Monte-Carlo algorithm: Pick random P on E_A and check $[p+1]P = \infty$.[1]

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Quantum Security

Original proposal in 2018 paper: $\mathbb{F}_p \approx 512$ bits.

- The exact cost of the Kuperberg/Regev/CJS attack is subtle – it depends on:
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- For fastest variant of Kuperberg, total cost of CSIDH-512 attack is at least 2^{56} qubit operations.
- Overheads from error correction, high quantum memory etc., not yet understood.

Venturing beyond the CSIDH

A selection of advances since original publication (2018):

- CSURF [CD19]: exploiting 2-isogenies.
- sqrtVelu [BDLS20]: square-root speed-up on computation of large-degree isogenies.
- Radical isogenies [CDV20]: significant speed-up on isogenies of small-ish degree.
- Some work on different curve forms (e.g. Edwards, Huff).
- Knowledge of $\mathrm{End}(E_0)$ and $\mathrm{End}(E_A)$ breaks CSIDH in classical polynomial time [Wes21].
- The SQALE of CSIDH [CCJR22]: carefully constructed CSIDH parameters less susceptible to Kuperberg's algorithm.
- CTIDH [$\mathrm{B}^2\mathrm{C}^2\mathrm{LMS}^2$]: Efficient constant-time CSIDH-style construction.

Now:

SIDH

Supersingular Isogeny Diffie–Hellman

Diffie-Hellman: High-level view

$$\begin{tikzcd}
g \arrow[r] \arrow[d] & g^{a} \arrow[d] \\
g^{b} \arrow[r] & g^{a*b}
\end{tikzcd}$$

SIDH: High-level view

$$
\begin{tikzcd}
E \arrow[r, "\varphi_A"] \arrow[d, "\varphi_B"'] & E/A \arrow[d, "\varphi_{B'}"] \\
E/B \arrow[r, "\varphi_{A'}"'] & E/\langle A, B \rangle
\end{tikzcd}
$$

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- Alice computes $\varphi_A : E \to E/A$; Bob computes $\varphi_B : E \to E/B$.

SIDH: High-level view

$$\begin{array}{ccc} E & \xrightarrow{\varphi_A} & E/A \\ {\scriptstyle\varphi_B}\downarrow & & \downarrow{\scriptstyle\varphi_{B'}} \\ E/B & \xrightarrow[\varphi_{A'}]{} & E/\langle A, B\rangle \end{array}$$

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- They both compute the shared secret

$$(E/B)/A' \cong E/\langle A, B\rangle \cong (E/A)/B'.$$

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Previous slide: "Alice somehow obtains $A' := \varphi_B(A)$."

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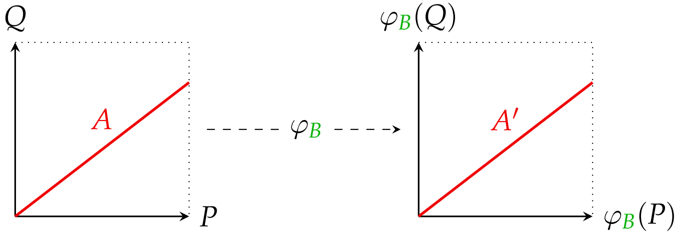

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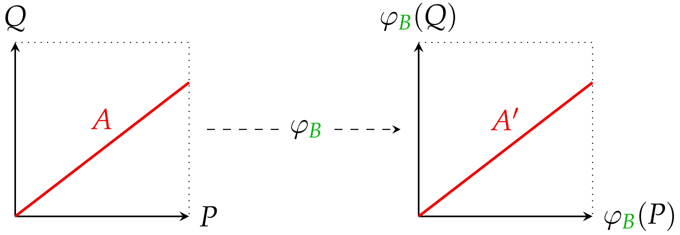

- Alice picks A as $\langle P + [a]Q \rangle$ for fixed public $P, Q \in E$.
- Bob includes $\varphi_B(P)$ and $\varphi_B(Q)$ in his public key.

$\Longrightarrow$ Now Alice can compute A' as $\langle \varphi_B(P) + [a]\varphi_B(Q) \rangle$!

SIDH in one slide

Public parameters:

- a large prime $p = 2^n 3^m - 1$ and a supersingular $E/\mathbb{F}_p$
- bases (P_A, Q_A) and (P_B, Q_B) of $E[2^n]$ and $E[3^m]$

Alice	public	Bob
$a \xleftarrow{\text{random}} \{0 \ldots 2^n - 1\}$		$b \xleftarrow{\text{random}} \{0 \ldots 3^m - 1\}$
$A := \langle P_A + [a] Q_A \rangle$ compute $\varphi_A : E \to E/A$		$B := \langle P_B + [b] Q_B \rangle$ compute $\varphi_B : E \to E/B$
$E/A,\ \varphi_A(P_B),\ \varphi_A(Q_B)$	$\longrightarrow$ $\longleftarrow$	$E/B,\ \varphi_B(P_A),\ \varphi_B(Q_A)$
$A' := \langle \varphi_B(P_A) + [a] \varphi_B(Q_A) \rangle$ $s := j\big((E/B)/A'\big)$		$B' := \langle \varphi_A(P_B) + [b] \varphi_A(Q_B) \rangle$ $s := j\big((E/A)/B'\big)$

Break it by: given public info, find secret key–φ_A or just A.

Security

Hard Problem:
Given

- supersingular public elliptic curves $E_0/\mathbb{F}_{p^2}$ and $E_A/\mathbb{F}_{p^2}$ connected by a secret 2^n-degree isogeny $\varphi_A : E_0 \to E_A$, and
- the action of φ_A on the 3^m-torsion of E_0,

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- Knowledge of $\mathrm{End}(E_0)$ and $\mathrm{End}(E_A)$ is sufficient to efficiently break it.
- Active attacker can recover secret.
- In SIDH, $\mathrm{End}(E_0)$ is fixed and $3^m \approx 2^n \approx \sqrt{p}$.
- If $3^m > 2^n$ or $3^m, 2^n > \sqrt{p}$, security claims are weakened.

Security of SIKE

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- No commutative group action to exploit here*

What about signatures?

CSI-FiSh (S '06, D-G '18, Beullens-Kleinjung-Vercauteren '19)

Identification scheme from $H \times S \to S$:

Prover	**Public**	**Verifier**
	$E \in S, \mathfrak{l}_i \in H$	
$s_i \leftarrow \$\, \mathbb{Z}$		
$\mathsf{sk} = \prod \mathfrak{l}_i^{s_i}$,		
$\mathsf{pk} = \mathsf{sk} * E \xrightarrow{\mathsf{pk}}$	pk	
	$\xleftarrow{c}$	$c \leftarrow \$\, \{0, 1\}$
$t_i \leftarrow \$\, \mathbb{Z}$		
$\mathsf{esk} = \prod \mathfrak{l}_i^{t_i}$,		
$\mathsf{epk}_1 = \mathsf{esk} * E$,		
$\mathsf{epk}_2 = \mathsf{esk} \cdot \mathsf{sk}^{-c}$	$\xrightarrow{\mathsf{pk}, \mathsf{epk}_1, \mathsf{epk}_2}$	check:
		$\mathsf{epk}_1 = \mathsf{epk}_2 * ([\mathsf{sk}^c] * E)$.

After k challenges c, an imposter succeeds with prob 2^{-k}.

SQISign (De Feo-Kohel-Leroux-Petit-Wesolowski '20)

Hard Problem in CSIDH, CSI-FiSh, etc:
Given elliptic curves E and $E' \in S$, find $\mathfrak{a} \in H$ such that
$\mathfrak{a} * E = E'$.

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public, secret, ephemeral secret, public challenge, public proof

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- SQISign ‘20 Digital signature. Small, slow, clean security assumption, no known attack avenues.

Thank you!

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