How to not break SIDH :(

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What is SIDH?

Recall: SIDH as an isogeny graph

- **Vertices**: j-invariants of elliptic curves defined over $\overline{\mathbb{F}_p}$.
- **Edges**: 2- and 3-isogenies of elliptic curves (up to some equivalence).

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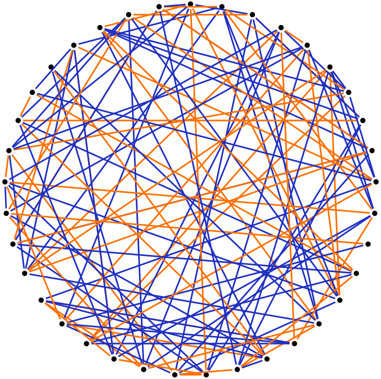

2 and 3-isogenies of elliptic curves over $\mathbb{F}_{431^2}$

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- A point $P \in E[m]$ is called an m-torsion point.
- The group $G = \langle P \rangle$ generated by an m-torsion point $P \in E[m]$ is the kernel of an m-isogeny written

$$f : E \to E/G.$$

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Break it by: given public info, find secret key: φ_A (or just A).

Here's some things that don't break it...

Extra points

Aim: given points P_B, Q_B on E, the image E/A of the secret isogeny $\varphi_A : E \to E/A$, and the images $\varphi_A(P_B)$ and $\varphi_B(Q_B)$, find φ_A.

Fact: with the parameters used in SIDH, the images $\varphi_A(P_B)$ and $\varphi_B(Q_B)$ uniquely determine the secret isogeny φ_A.

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$\rightsquigarrow$ can't even write down the result without decomposing into a sequence of smaller-degree maps.

- No known algorithms for interpolating and decomposing at the same time.

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$$E(\mathbb{F}_{p^2}) \cong (\mathbb{Z}/2^n)^2 \times (\mathbb{Z}/3^m)^2 .$$

$\Longrightarrow$ can't learn anything about 2^n from 3^m using groups alone.
(Annoying: This shows up in many disguises.)

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- Idea: Find an appropriate endomorphism τ of degree $3^m r$; recover 3^m-part as above; brute-force the $r$emaining part.
  $\rightsquigarrow$ image of r-torsion point under φ_A
  $\Longrightarrow$ (details) $\Longrightarrow$ Recover the secret φ_A.

$\ddot{\frown}$ To get r small enough to be an attack, we have to change the SIDH parameters so that Alice's isogeny has a much higher degree than Bob's.

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- Life sucks.

  ¯_(ツ)_/¯

The pure isogeny problem

Fundamental problem: given supersingular E and $E'/\mathbb{F}_{p^2}$ that are ℓ^n-isogeneous, compute an isogeny $\phi : E \to E'$.

The pure isogeny problem

Example

Choose

$$E/\mathbb{F}_{431} : y^2 = x^3 + 1 \quad \text{and} \quad E'/\mathbb{F}_{431} : y^2 = x^3 + 291x + 298.$$

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These elliptic curves are $2^2 = 4$-isogenous. Problem: compute an isogeny $f : E \to E'$.

The kernel of $f : E \to E'$ is generated by a point $P \in E(\overline{\mathbb{F}_p})$ of order 4.

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Solution (b) is meet-in-the-middle: complexity $\tilde{O}(p^{1/4})$.

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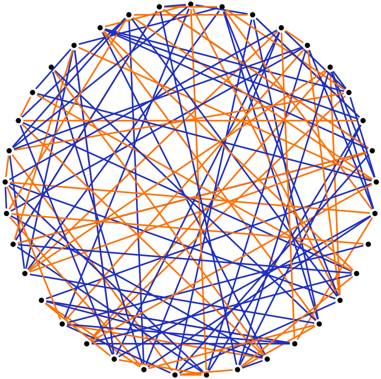

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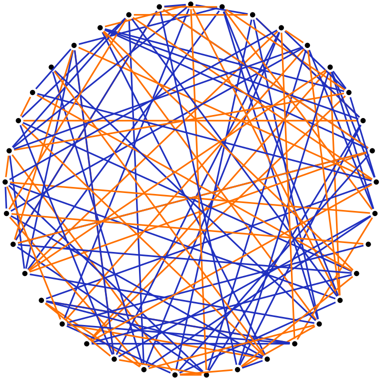

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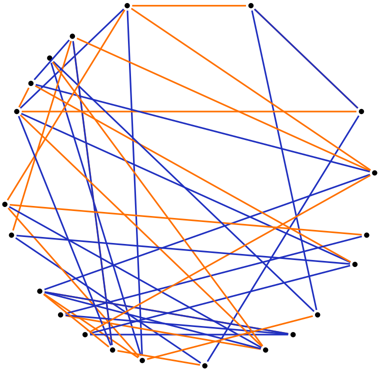

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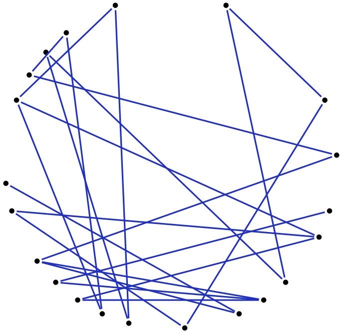

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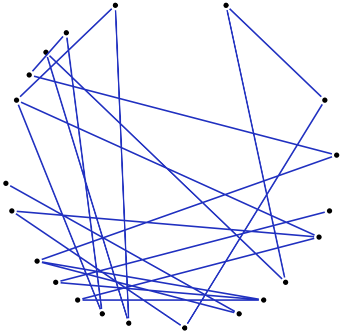

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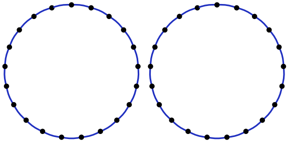

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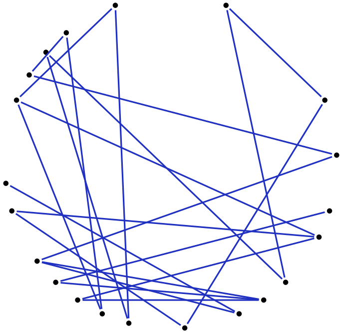

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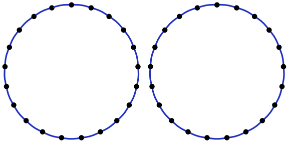

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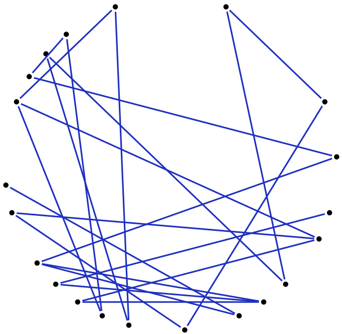

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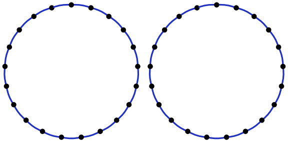

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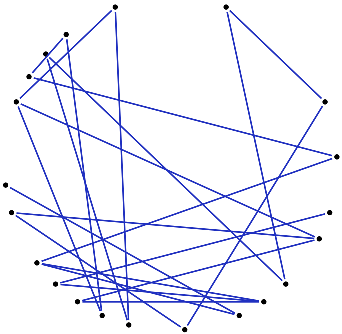

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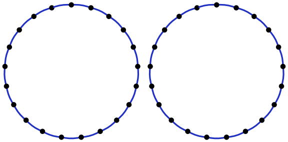

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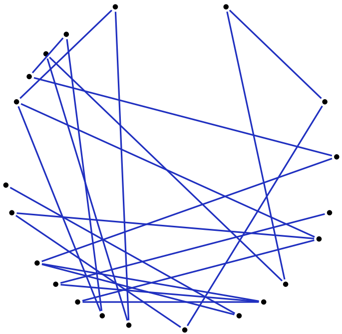

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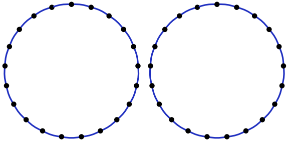

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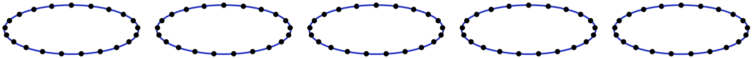
(Delfs-Galbraith, Biasse-Jao-Sankar)

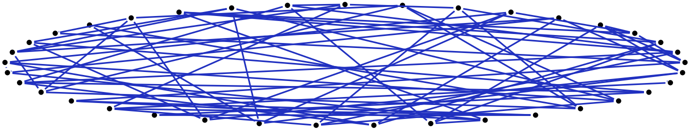
More graphs defined over $\mathbb{F}_p$



From 1-dimensional $E/\mathbb{F}_{p^2}$,

construct 2-dimensional $W(E)/\mathbb{F}_p$

'Weil restriction'



This picture is very unlikely to be accurate.

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More equivalent categories: lifting to $\mathbb{C}$

$$\left\{ \begin{array}{c} \text{Elliptic curves } E \text{ defined over } \mathbb{C} \\ \text{with } \operatorname{End}(E) = R \end{array} \right\}$$

Here computing isogenies is easy!

$$\updownarrow$$

$$\left\{ \begin{array}{c} \text{Non-supersingular elliptic curves defined over } \mathbb{F}_q \\ \text{with } \operatorname{End}(E) = R \end{array} \right\}$$

Here computing isogenies is harder.

More equivalent categories: lifting to $\mathbb{C}$

A well-chosen subset of

$$\left\{ \begin{array}{c} \text{Elliptic curves } E \text{ defined over } \mathbb{C} \\ \text{with } \phi \in \mathrm{End}(E) \end{array} \right\}$$

Here computing isogenies is easy!

$$\updownarrow$$

$$\left\{ \begin{array}{c} \text{Supersingular elliptic curves defined over } \mathbb{F}_q \\ \text{with non-scalar } \phi \in \mathrm{End}(E) \end{array} \right\}$$

Here computing isogenies is harder.

More equivalent categories: lifting to $\mathbb{C}$

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$$\left\{ \begin{array}{c} \text{Elliptic curves } E \text{ defined over } \mathbb{C} \\ \text{with } \phi \in \mathrm{End}(E) \end{array} \right\}$$

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- Computing the equivalence is slow.

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Here computing isogenies is harder.

- Computing the equivalence is slow.
- Finding a non-scalar endomorphism is hard.

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A well-chosen subset of

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Here computing isogenies is harder.

- Computing the equivalence is slow.
- Finding a non-scalar endomorphism is hard.
- If you can find non-scalar endomorphisms, SIDH is probably already broken by earlier work (Kohel-Lauter-Petit-Tignol and Galbraith-Petit-Shani-Ti).

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Thank you!