Isogeny-based cryptography: why, how, and the latest news

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- Rich mathematical structure $\rightsquigarrow$ most flexible* post-quantum applications. Since 2018:
  - Only pq non-interactive key exchange (c.f. Diffie-Hellman)
  - Two different signature schemes
  - Oblivious pseudorandom functions
  - Threshold schemes
  - ElGamal-style message encryption
  - ...

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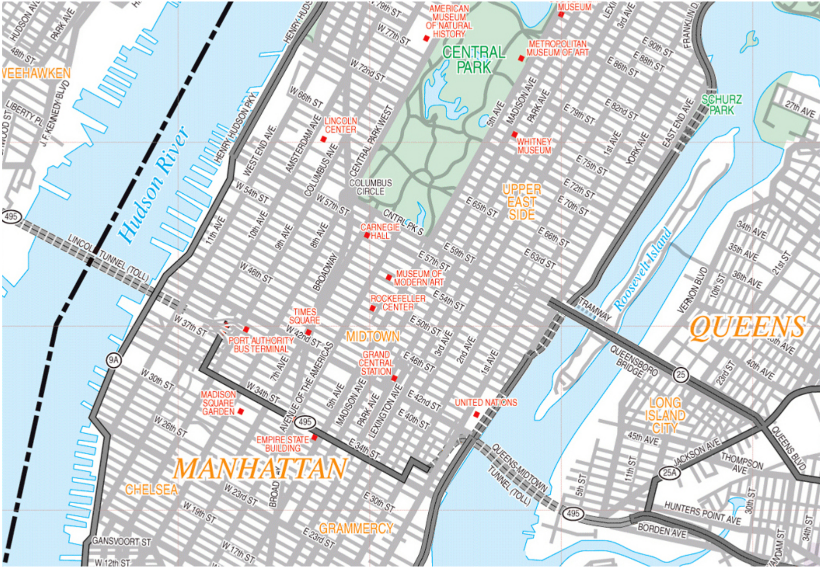
- Lowest memory, most flexible Hard Problem admits a subexponential quantum attack, the complexity of which is still an active research topic.
  - Difficult to make concrete parameter choices.
- Slow: Fastest key encapsulation is $\approx \times 25$ slower than ECC or the fastest pq option (lattices).

Isogeny-based cryptography: how?

- Hard Problems in isogeny-based cryptography are (mostly) based on elliptic curves.
- On a high level, this can be abstracted away…

Graph walking Diffie–Hellman?



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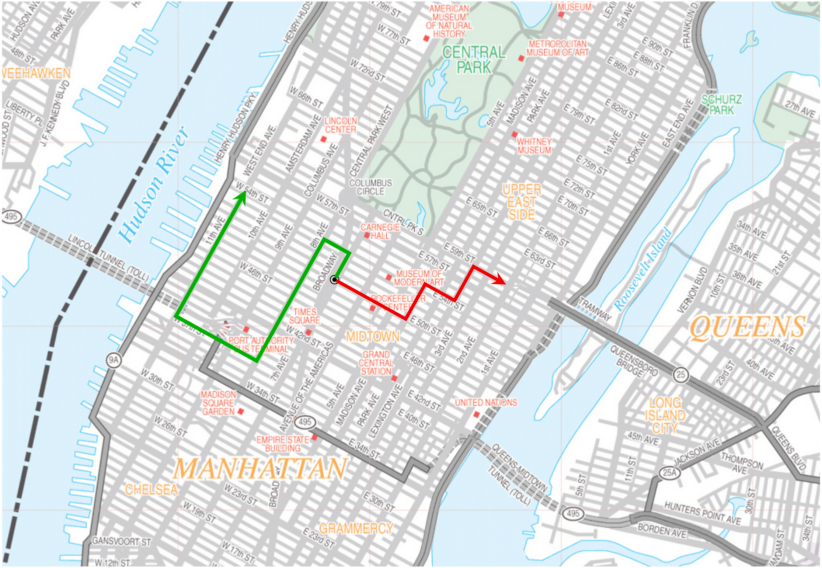

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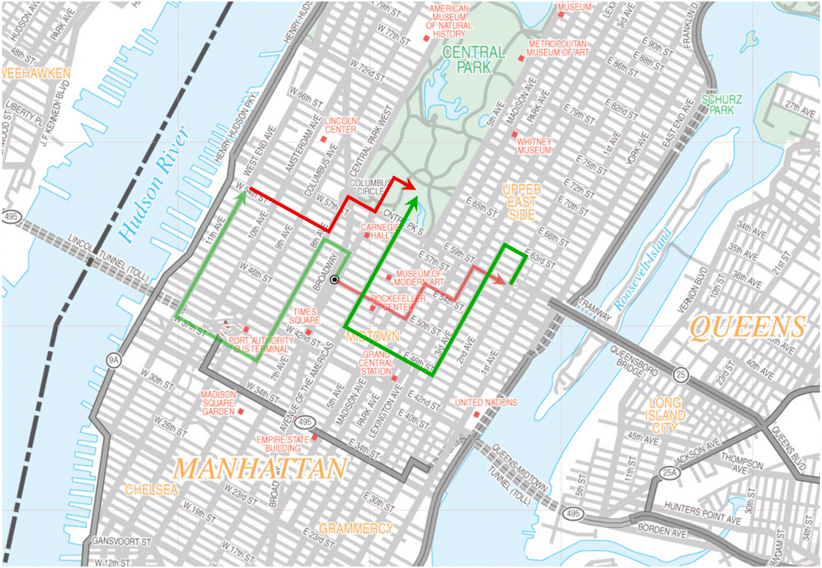

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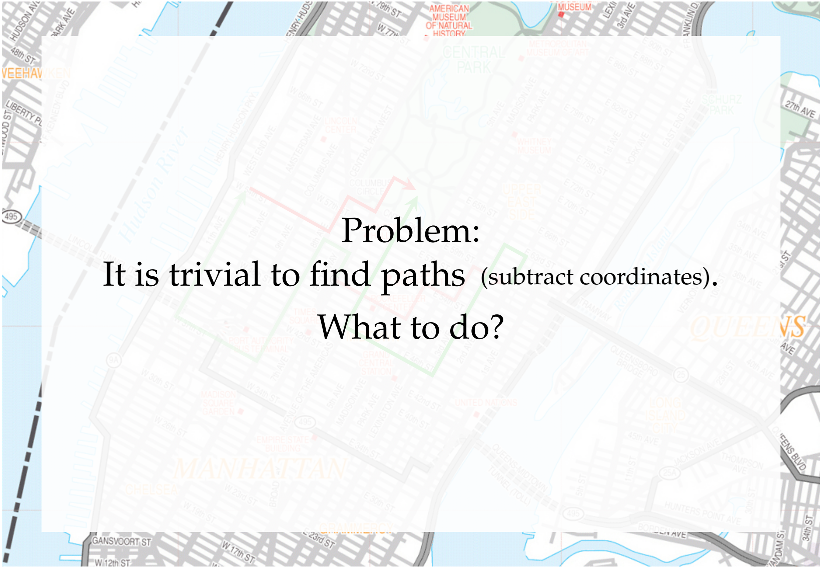

Problem:

It is trivial to find paths (subtract coordinates).

What to do?

Big picture

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It is easy to construct graphs that satisfy *almost* all of these — not enough for crypto!

Ex: CSIDH (Castryck-Lange-M.-Panny-Renes '18)

Traditionally, Diffie-Hellman works in a group G via the map

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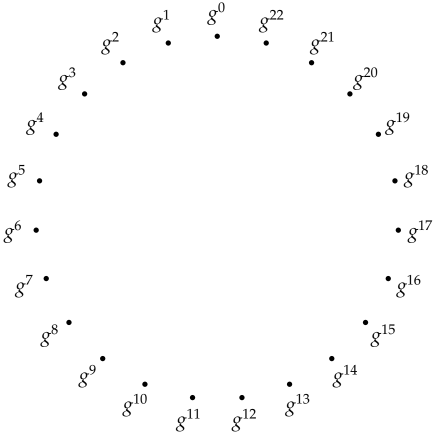
$\rightsquigarrow$ Idea:

Replace exponentiation on the group G by a group action of a group H on a set S:

$$H \times S \to S.$$

Square-and-multiply

Suppose $G \cong \mathbb{Z}/23$ and that Alice computes g^{13}.



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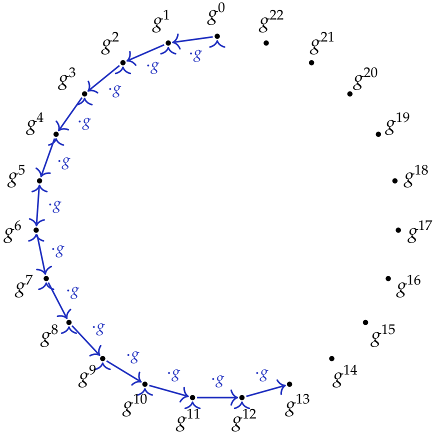

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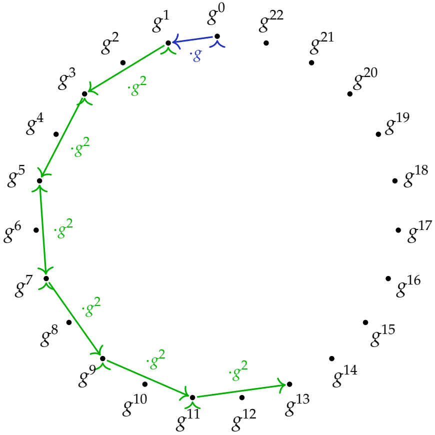

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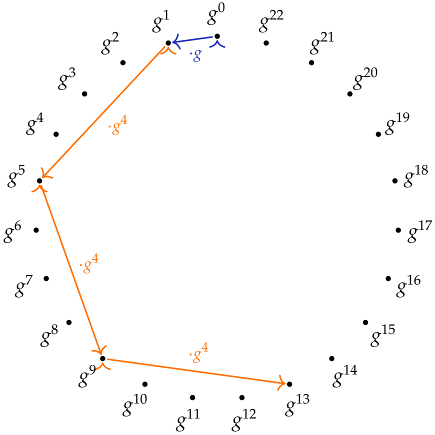

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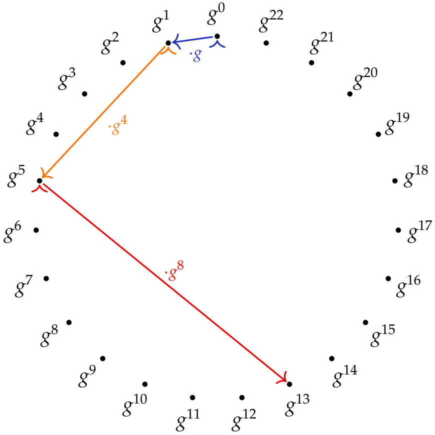

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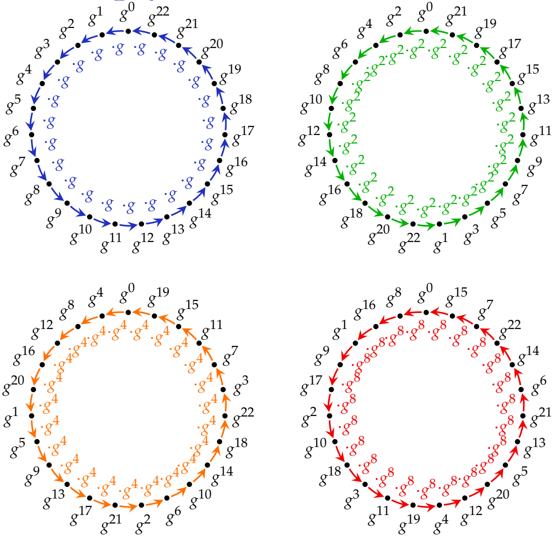

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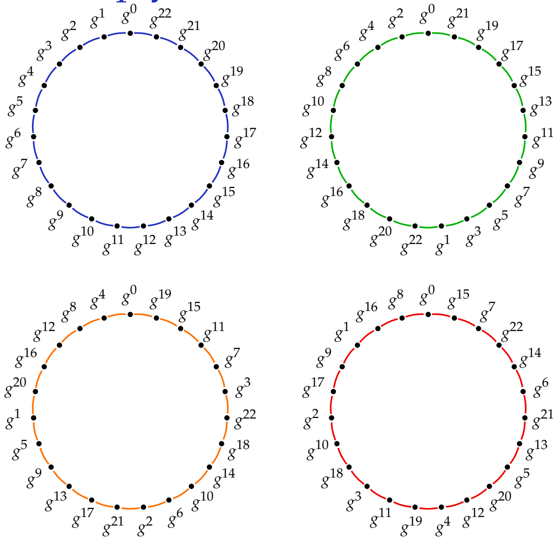

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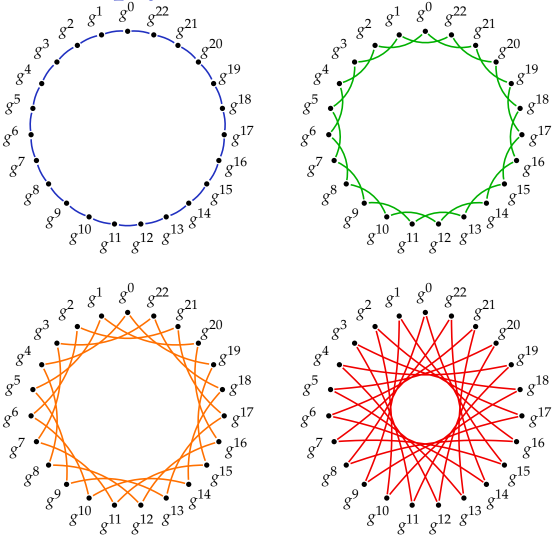

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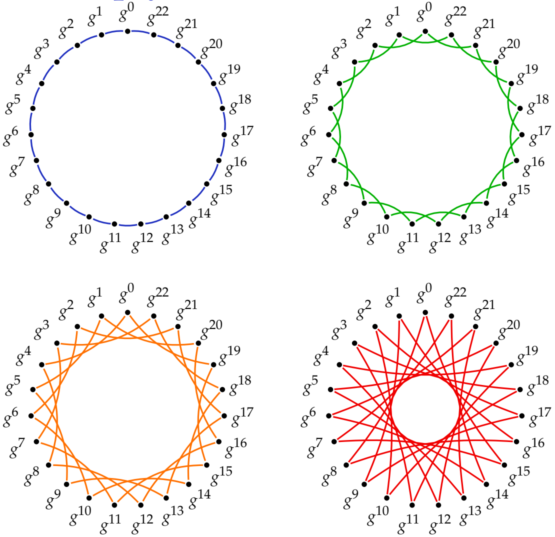
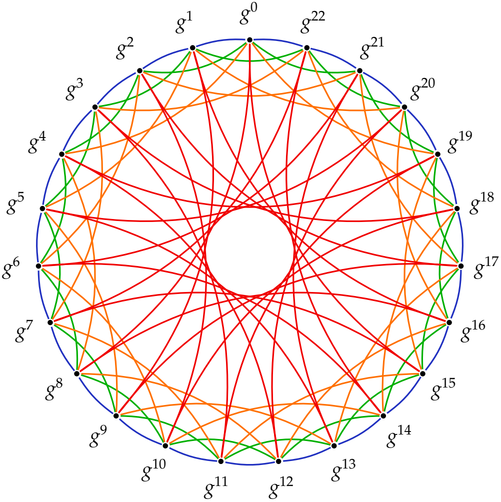


Cycles are compatible: [right, then left] = [left, then right], etc.

Union of cycles: rapid mixing



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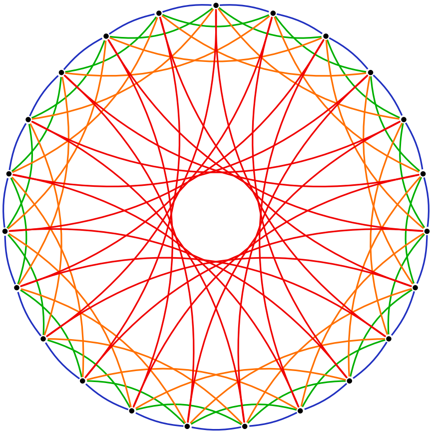
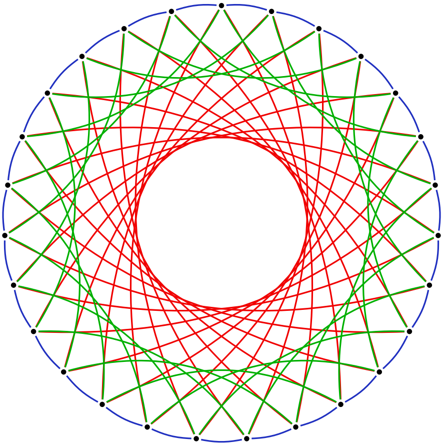


CSIDH: Nodes are now elliptic curves and edges are isogenies.

Graphs of elliptic curves



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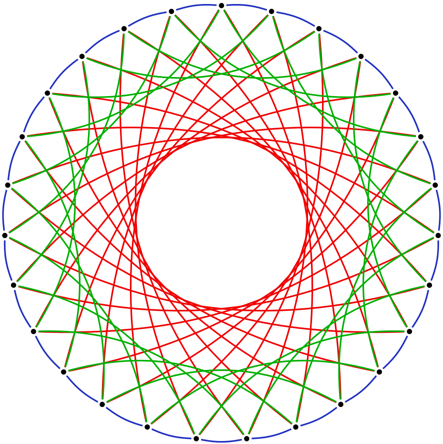

Nodes: Supersingular curves $E_A\colon y^2 = x^3 + Ax^2 + x$ over $\mathbb{F}_{419}$.

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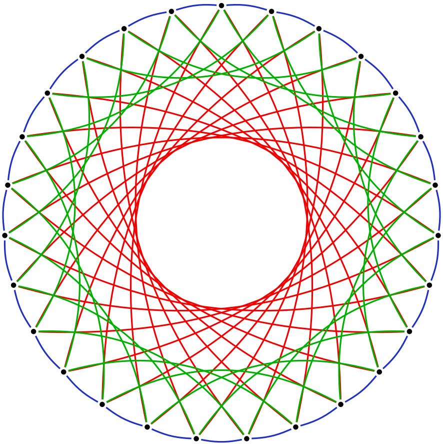

Nodes: Supersingular curves $E_A : y^2 = x^3 + Ax^2 + x$ over $\mathbb{F}_{419}$.
Edges: 3-, 5-, and 7-isogenies.

Quantumifying Exponentiation

- We want to replace the exponentiation map

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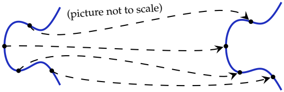
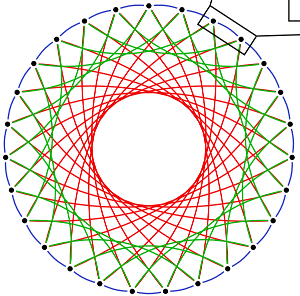
Graphs of elliptic curves

A 3-isogeny



$$E_{51}\colon y^2 = x^3 + 51x^2 + x \longrightarrow E_9\colon y^2 = x^3 + 9x^2 + x$$

$$(x, y) \longmapsto \left(\frac{97x^3 - 183x^2 + x}{x^2 - 183x + 97},\; y \cdot \frac{133x^3 + 154x^2 - 5x + 97}{-x^3 + 65x^2 + 128x - 133} \right)$$



Diffie and Hellman go to the CSIDH

Alice

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Bob

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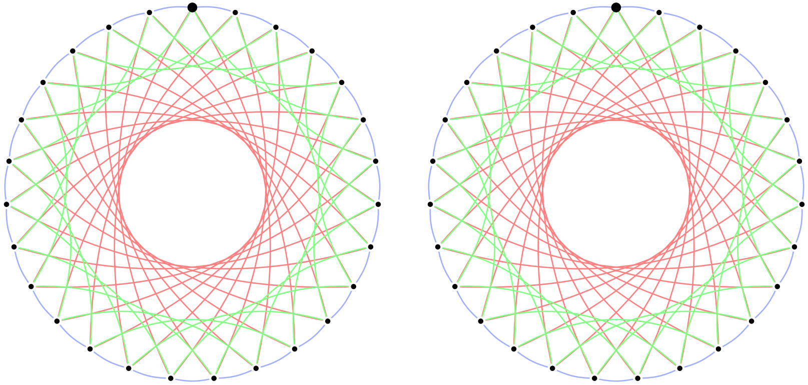

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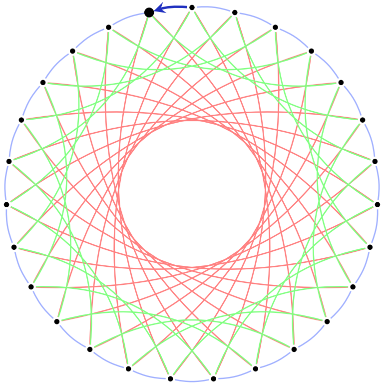

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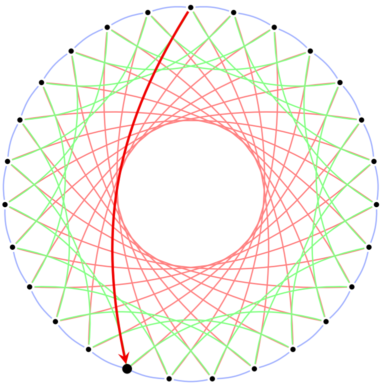

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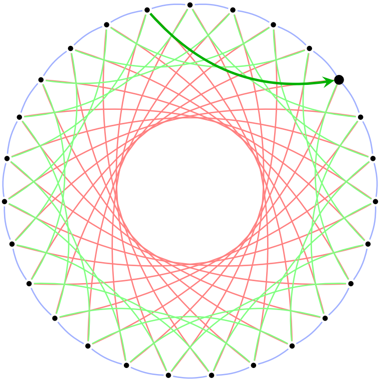

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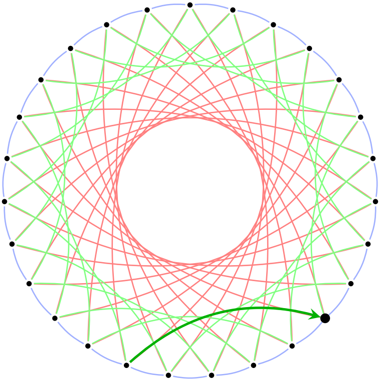

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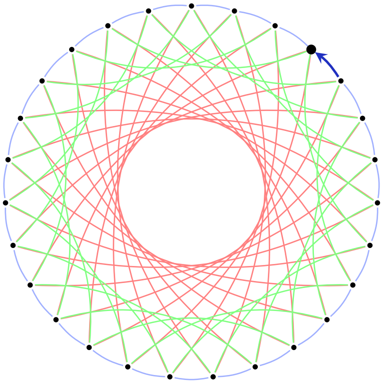

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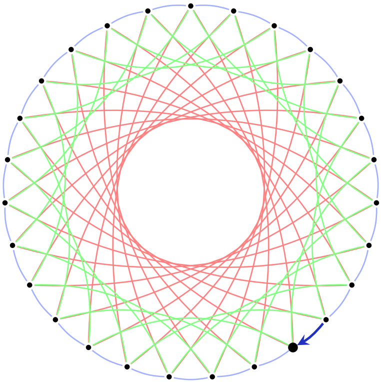

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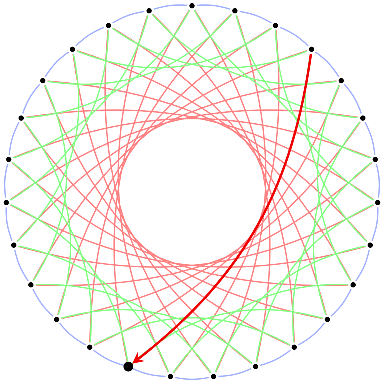

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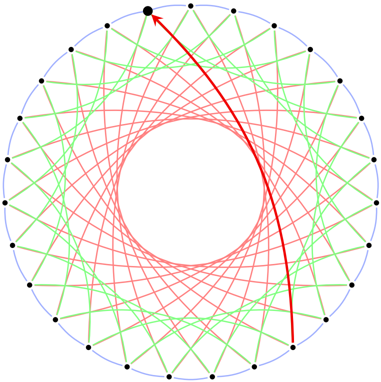

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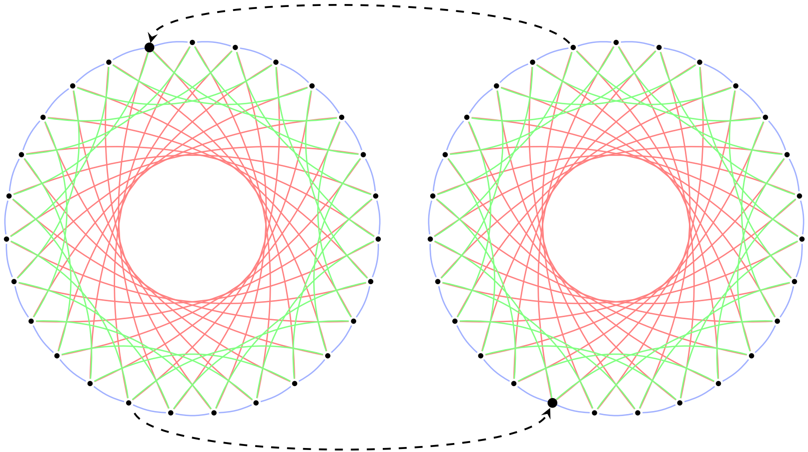

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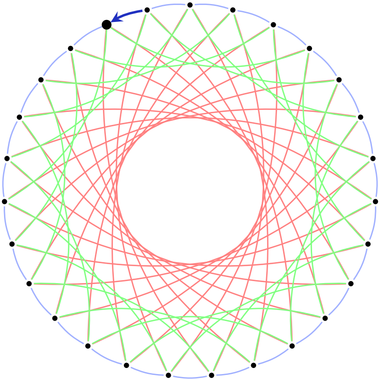

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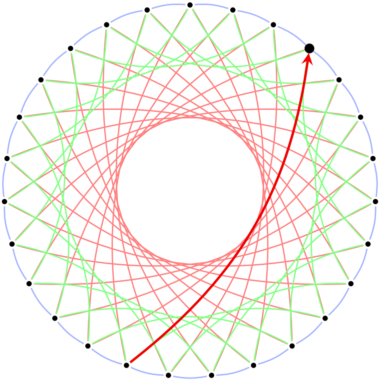

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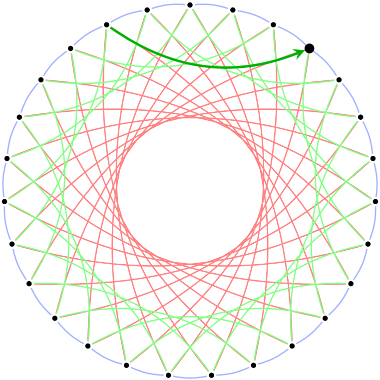

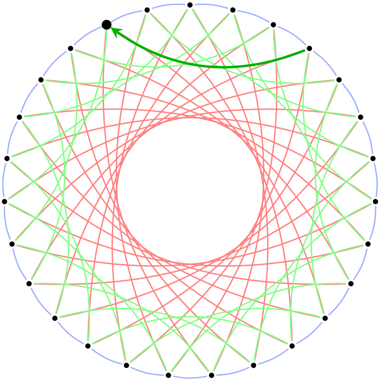

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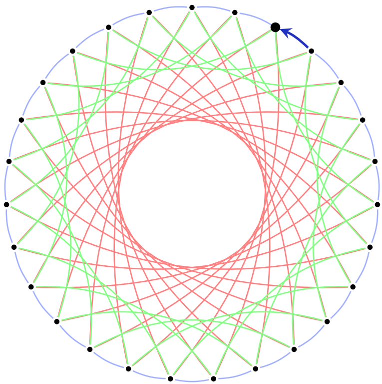

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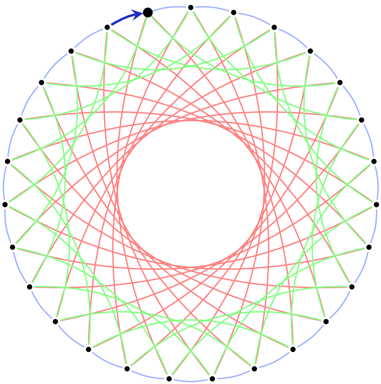

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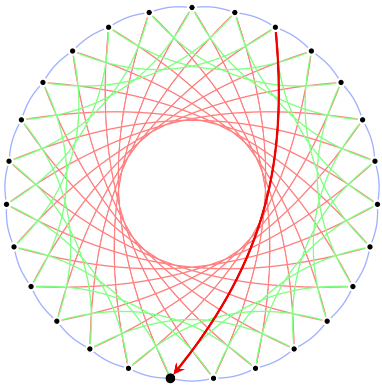

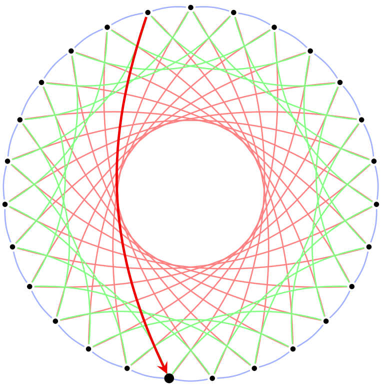

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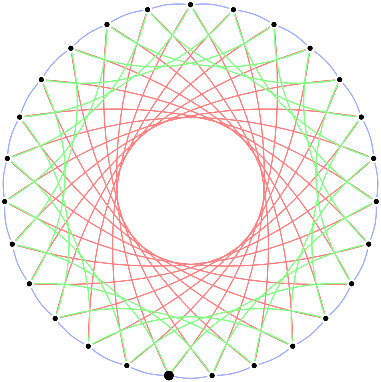

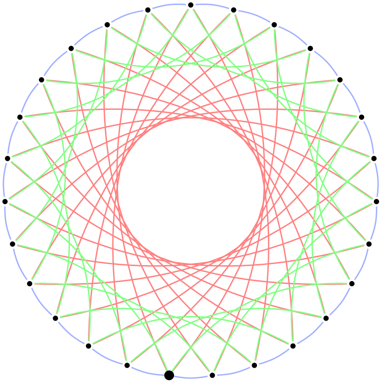

Ex: CSI-FiSh (S '06, D-G '18, Beullens-Kleinjung-Vercauteren '19)

Identification scheme from $H \times S \to S$:

Prover		**Public**		**Verifier**
		$E \in S, \mathfrak{l}_i \in H$		
$s_i \leftarrow \$ \, \mathbb{Z}$				
$\mathsf{sk} = \prod \mathfrak{l}_i^{s_i}$,				
$\mathsf{pk} = \mathsf{sk} * E$	$\xrightarrow{\mathsf{pk}}$	pk		
				$c \leftarrow \$ \, \{0, 1\}$
$t_i \leftarrow \$ \, \mathbb{Z}$	$\xleftarrow{c}$			
$\mathsf{esk} = \prod \mathfrak{l}_i^{t_i}$,				
$\mathsf{epk}_1 = \mathsf{esk} * E$,				
$\mathsf{epk}_2 = \mathsf{esk} \cdot \mathsf{sk}^{-c}$		$\xrightarrow{\mathsf{pk}, \mathsf{epk}_1, \mathsf{epk}_2}$		check:
				$\mathsf{epk}_1 = \mathsf{epk}_2 * ([\mathsf{sk}^c] * E)$.

After k challenges c, an imposter succeeds with prob 2^{-k}.

Ex: SQISign (De Feo-Kohel-Leroux-Petit-Wesolowski '20)

Hard Problem in CSIDH, CSI-FiSh, etc:
Given elliptic curves E and $E' \in S$, find $\mathfrak{a} \in H$ such that
$\mathfrak{a} * E = E'$.

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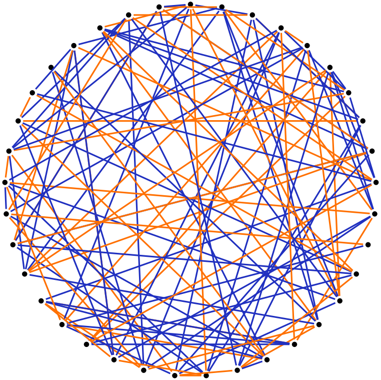
Main idea: Graph-walking Diffie-Hellman on this graph:

- **Vertices**: isomorphism classes of elliptic curves.
- **Edges**: 2- and 3-isogenies of elliptic curves (up to $\cong$).



2 and 3-isogenies of elliptic curves over $\mathbb{F}_{431^2}$

A new result (De Quehen-Kutas-Leonardi-M.-Panny-Petit-Stange)

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  - Backdoor primes and starting curves.

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- SQISign '20 Digital signature. Small, slow, clean security assumption, no known attack avenues.

Thank you!