Cryptography and quantum computers: Where do we stand?

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in association with
National Cyber Security Centre

What is this all about?

Cryptography

Sender — Channel with eavesdropper 'Eve' — Receiver

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Problems:

- Communication channels store and spy on our data
- Communication channels are modifying our data

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- Communication channels store and spy on our data
- Communication channels are modifying our data

Goals:

- Confidentiality despite Eve's espionage.
- Integrity: recognising Eve's espionage.

(Slide mostly stolen from Tanja Lange)

Post-quantum cryptography

Sender $\longrightarrow$ Channel with eavesdropper ‘Eve’ $\longrightarrow$ Receiver

Post-quantum cryptography

Sender → Channel with eavesdropper 'Eve' → Receiver

- Eve has a quantum computer.
- Harry and Meghan don't have a quantum computer.

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Main goal: replace the use of the discrete logarithm problem in asymmetric cryptography with something quantum-resistant.

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  Fastest encryption, huge keys, slow signatures.
- Multivariate signatures: based on solving simulateneous multivariate equations.
  Short signatures, large public keys, slow.

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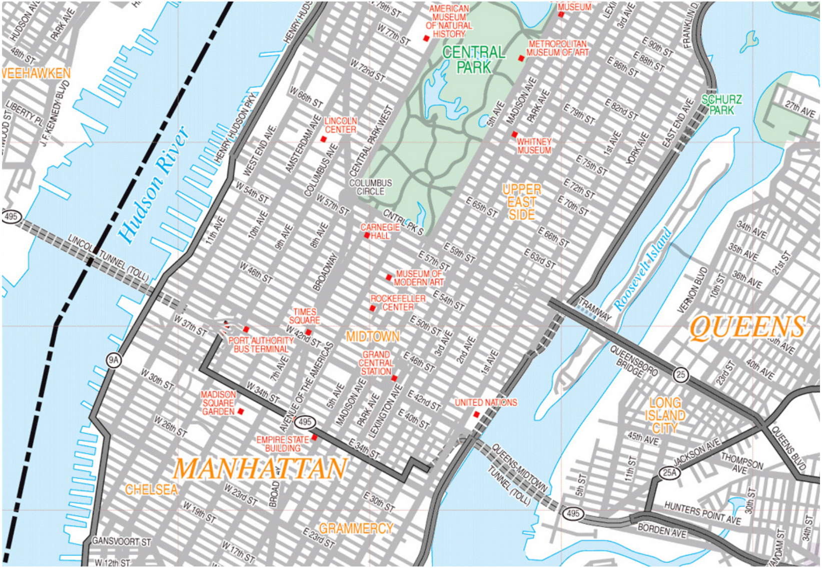
Case study: Isogenies. Graph walking Diffie-Hellman?



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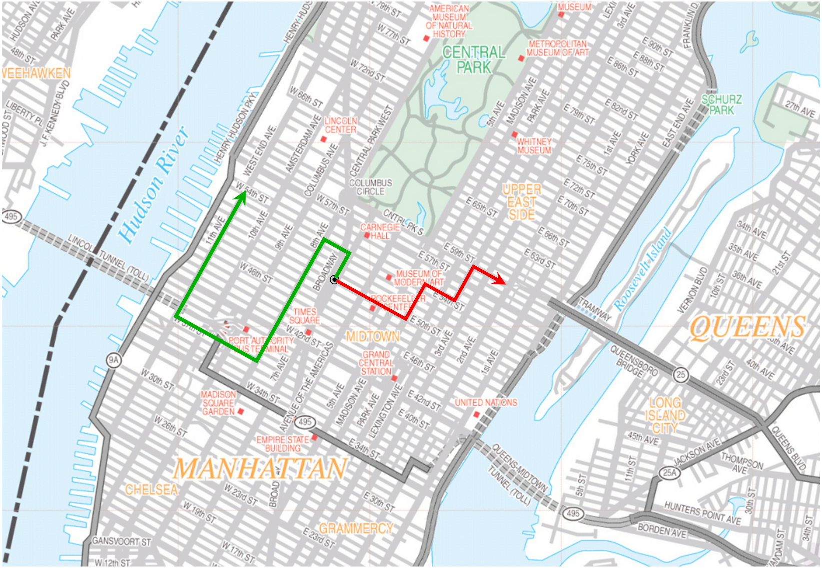

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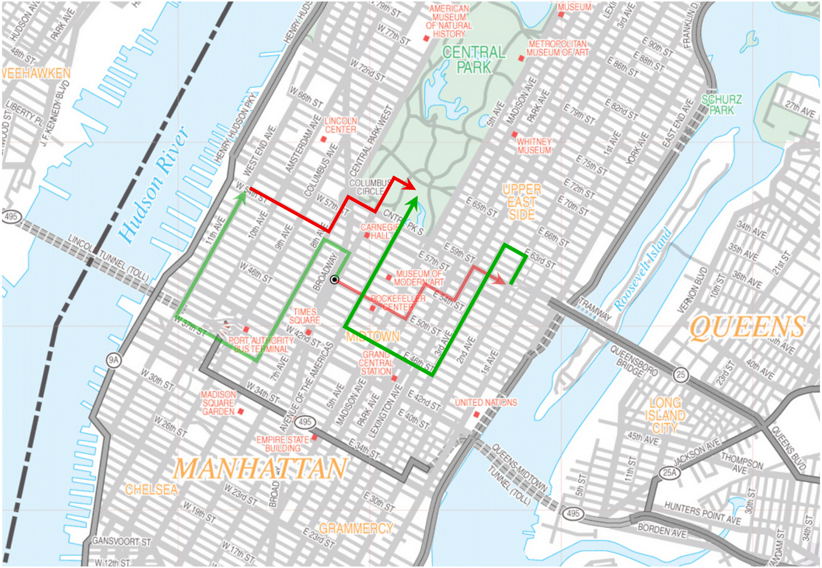

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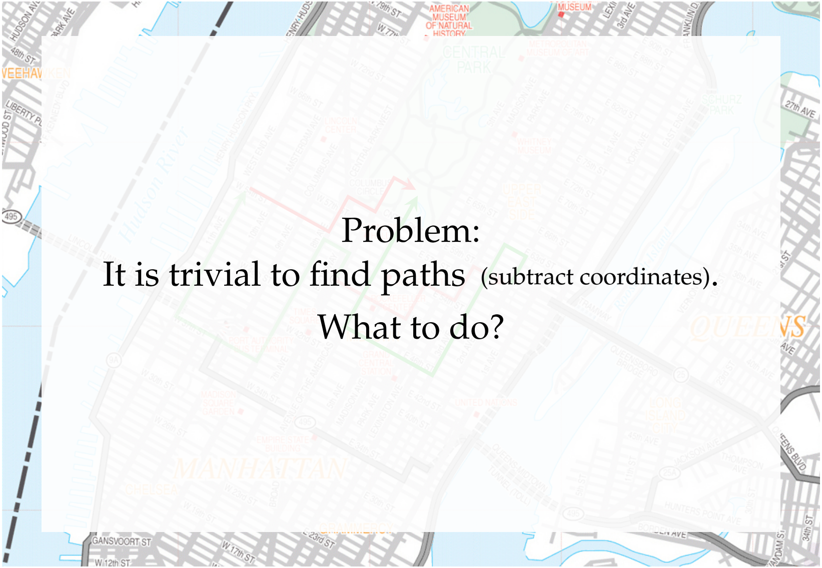

Problem:

It is trivial to find paths (subtract coordinates).

What to do?

Case study: Isogenies. Big picture

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- Isogenies are a source of exponentially-sized graphs.
- We can walk efficiently on these graphs.
- Fast mixing: short paths to (almost) all nodes.
- No known efficient algorithms to recover paths from endpoints.
- Enough structure to navigate the graph meaningfully.
  That is: some well-behaved 'directions' to describe paths.

Case study: Key exchange from isogenies

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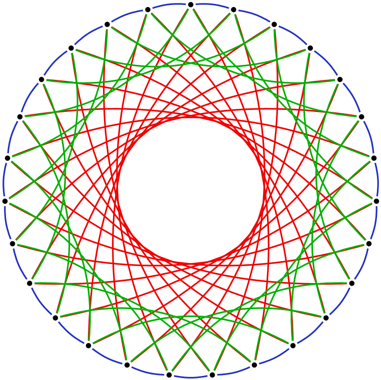

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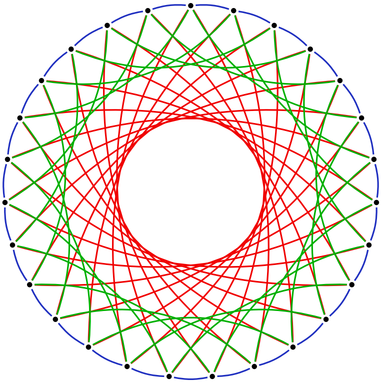

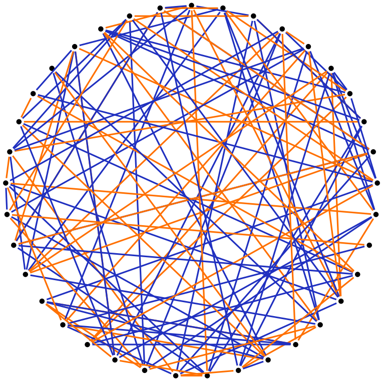

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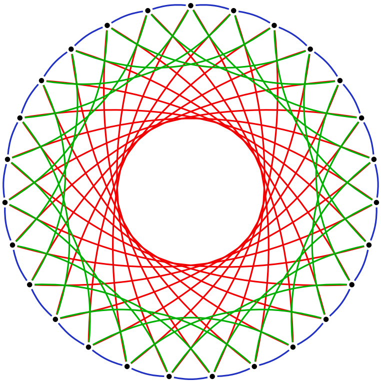
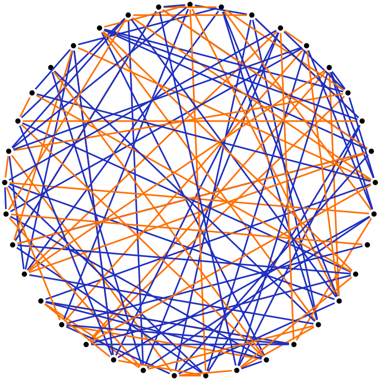
At this time, there are two families of systems:



CSIDH [ˈsiːˌsaɪd]
`https://csidh.isogeny.org`



SIKE
`https://sike.org`

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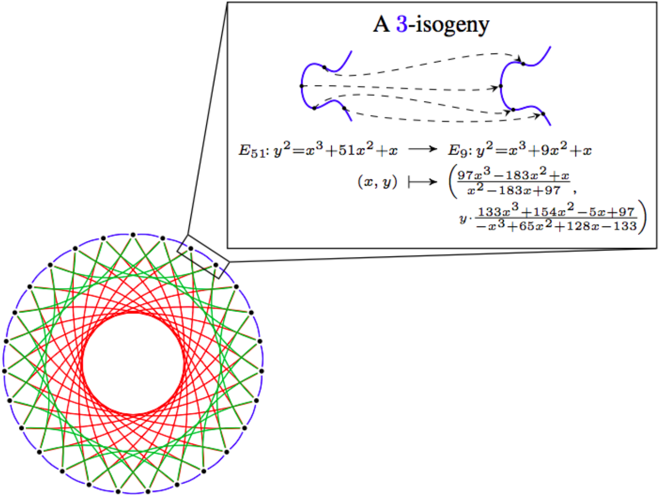
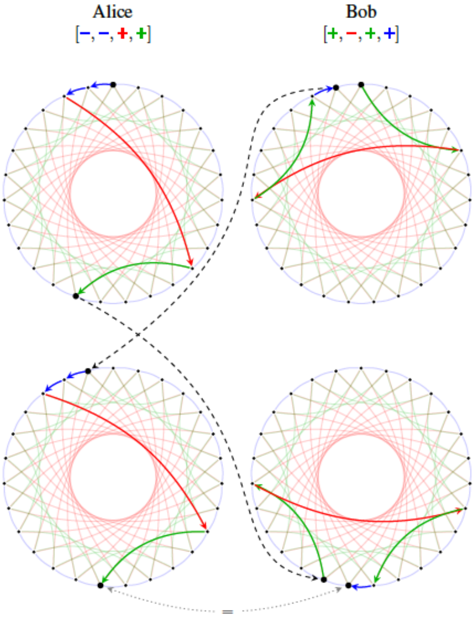


Case study: Isogenies. Key exchange at the CSIDH



Where are we now?

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- Post-quantum cryptography discussion dominated by NIST competition for standardization.
- This initiative comes after a US report with:

> **Key Finding 10:** Even if a quantum computer that can decrypt current cryptographic ciphers is more than a decade off, the hazard of such a machine is high enough—and the time frame for transitioning to a new security protocol is sufficiently long and uncertain—that prioritization of the development, standardization, and deployment of post-quantum cryptography is critical for minimizing the chance of a potential security and privacy disaster.

Where are we now (according to NIST)?

The NIST not-a-competition:

- Had 82 submissions in 2017.
- 69 were accepted.
- 15 submissions currently in 3rd round, aiming for a total of 4 rounds.
- Aiming for standardization in 2022.

Where are we now (according to NIST)?

Round 1 (2016):

	Signatures	KEM
Code-based	2	17
Hased-based	3	0
Isogeny-based	1	1
Lattice-based	5	21
Multivariate	7	2
Others	2	4

(Slide mostly stolen from Dustin Moody)

Where are we now (according to NIST)?

Round 3 (2020):

	Signatures	KEM
Code-based	0	3
Hased-based	2	0
Isogeny-based	0	1
Lattice-based	2	5
Multivariate	2	0

Where are we now

Round 3 (2020):

	Signatures	KEM
Code-based	0	3
Hased-based	2	0
Isogeny-based	0	1
Lattice-based	2	5
Multivariate	2	0

The field of isogeny-based is still developing. Since 2016:

2018 CSIDH, allowing for non-interative key exchange

2019 CSI-FiSh, efficient compact signatures based on CSIDH

2020 SQI-Sign, 'efficient' compact signatures

- Many more schemes building on the above

What can we do?

We have:

- KEM/Encryption and signatures
  (many options from NIST competition, also more options since).
- Diffie-Hellman-style / non-interactive key exchange
  (only option is with CSIDH).

We don't have:

- Anything else! For example, privacy-preserving protocols.

Important open problems/research directions

Needed for many post-quantum candidates:

- Thorough cryptanalysis – classical and quantum.
- Secure and efficient implementation (especially considering hardware limitations).
- Meaningful comparison between candidates (must come from comparable implementations).
- More advanced protocols.

Thank you!